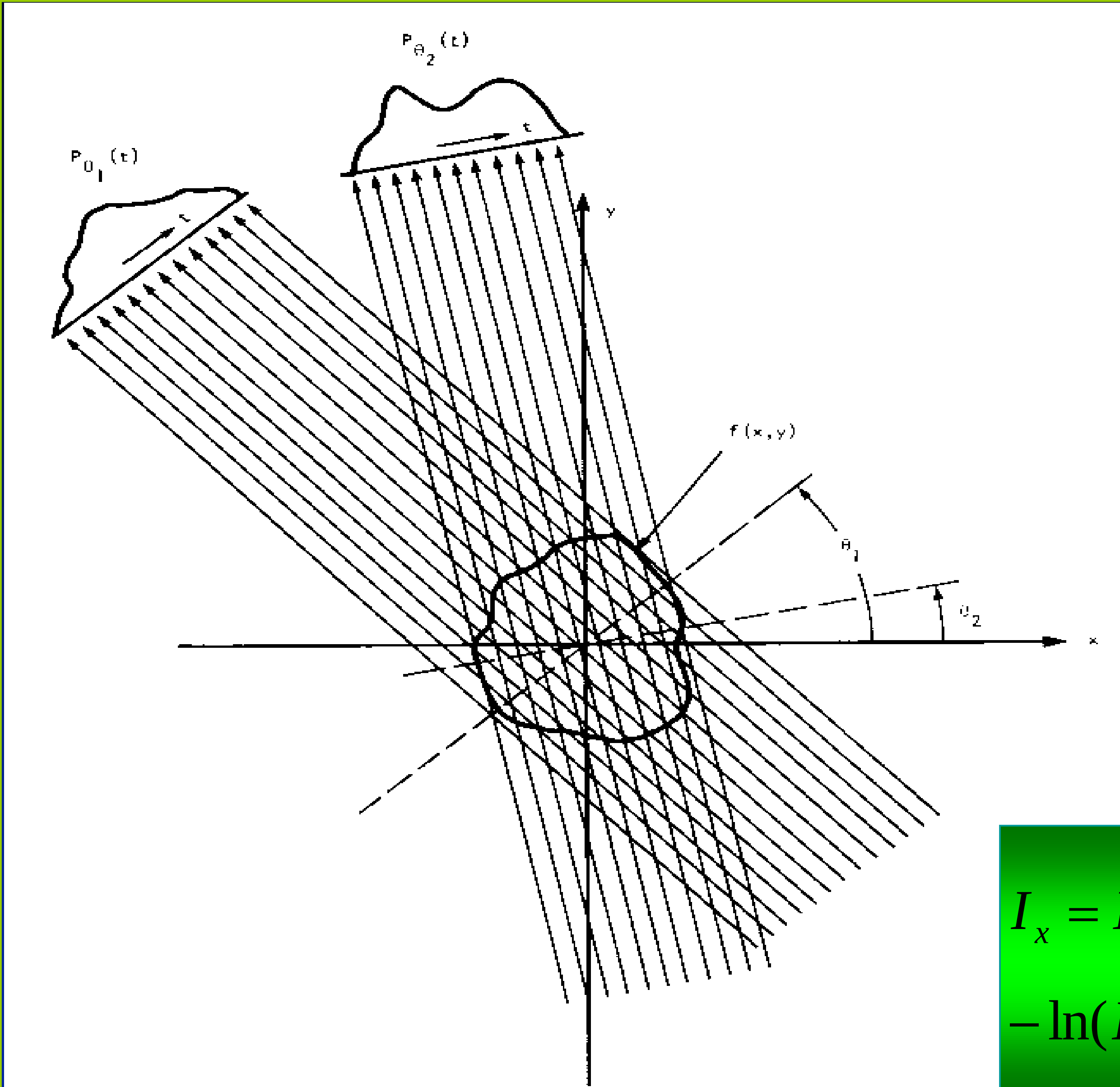
Reconstruction Technique

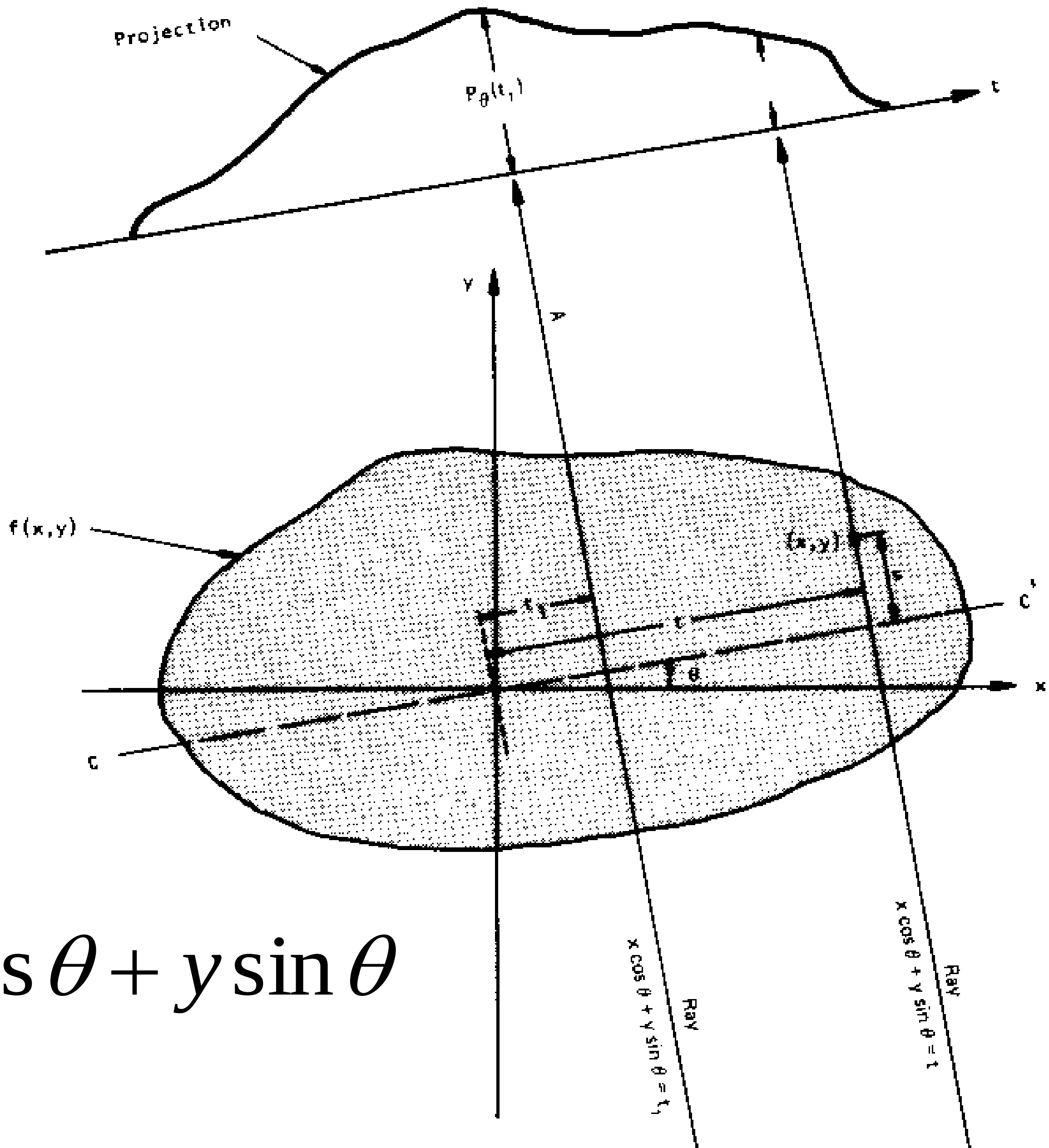
Parallel Projection



$$I_x = I_\circ e^{-\int \mu x} \Rightarrow$$

$$-\ln(I_\circ / I_x) = \int \mu x = pr$$

Projection in Cartesian and polar system:



$$l = x\cos\theta + y\sin\theta$$

Projection in Cartesian and polar system:

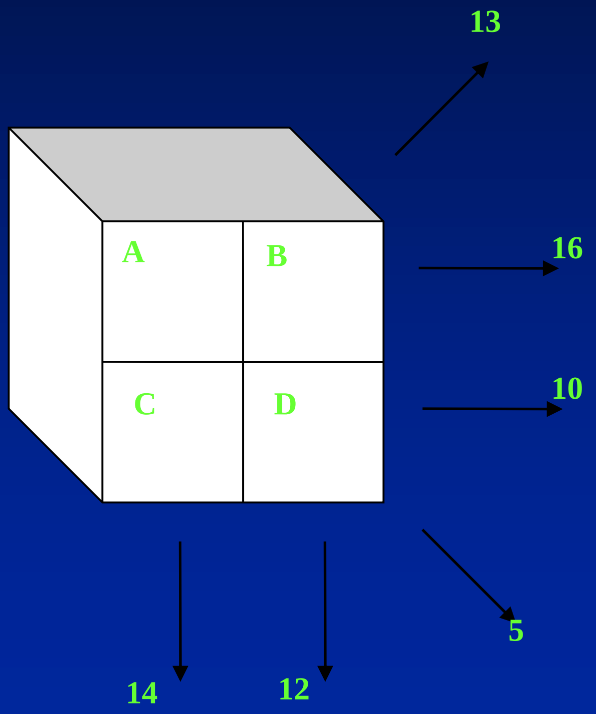
$$pr_\theta(\ell) = \int_{(\theta,\ell)line} f(x,y)ds$$

in the (t,s) coordinate:

$$\begin{bmatrix} \ell \\ s \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

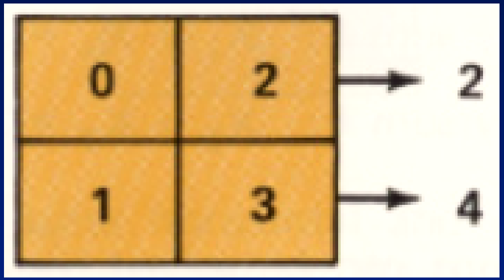
$$pr_\theta(\ell) = \int_{-\infty}^{\infty} f(\ell,s)ds$$

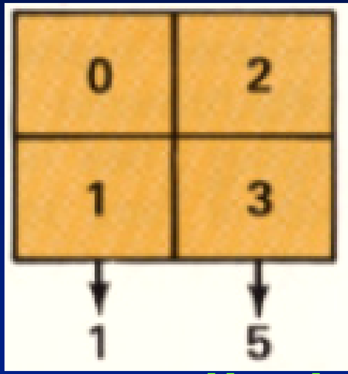
Algebraic (Iterative) Reconstruction Technique (ART)



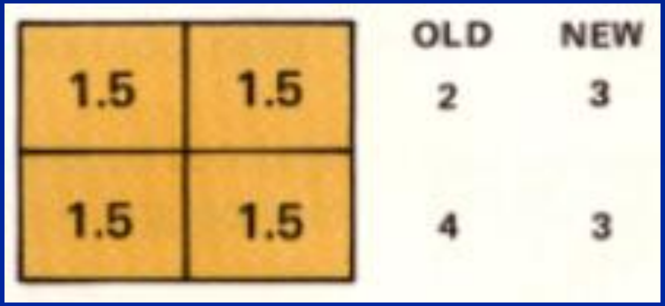
Algebraic (Iterative) Reconstruction Technique (ART)

We have originally output from detector:

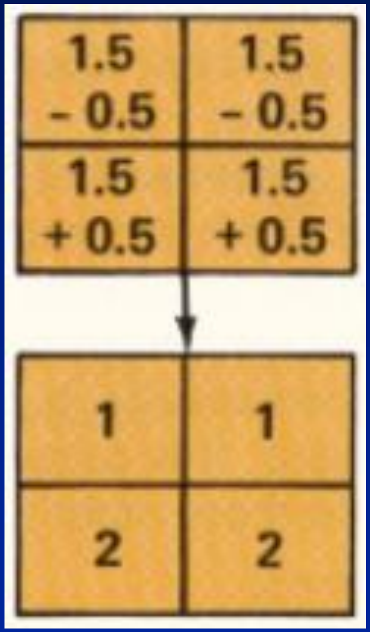


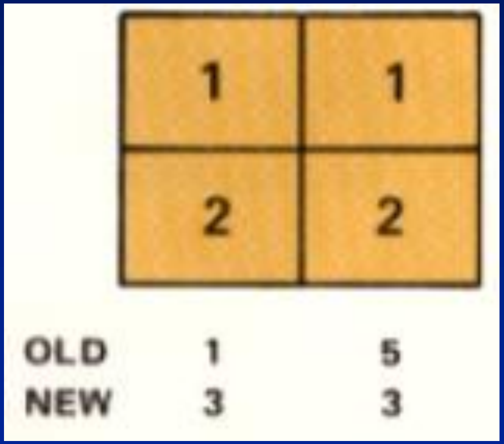


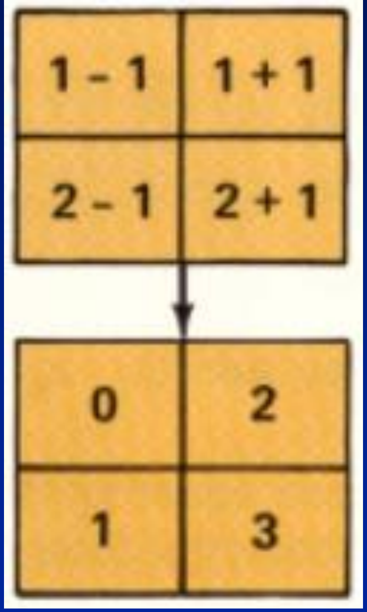
We start with the first prediction: 4/6=1.5



Algebraic (Iterative) Reconstruction Technique (ART)







ART Algorithm

For each actual projection prθ(l), predict pixel value and obtain predicted prθp(l) value

$$\mathrm{e}_\theta = \left[\mathrm{Pr}_\theta^p(\ell) - \mathrm{Pr}_\theta(\ell)\right] / \,\mathrm{N}_\theta(\ell)$$

$$f^{\,p+1}(x, y) = f^{\,p}(x, y) + e^p(\ell)$$

$$f^{\,p+1}(x, y) = f^{\,p}(x, y) \times e^p(\ell)$$

ART Algorithm

Where :

$$pr_{\theta}^{p}(\ell) = \sum w(x,y) f^{p}(x,y)$$

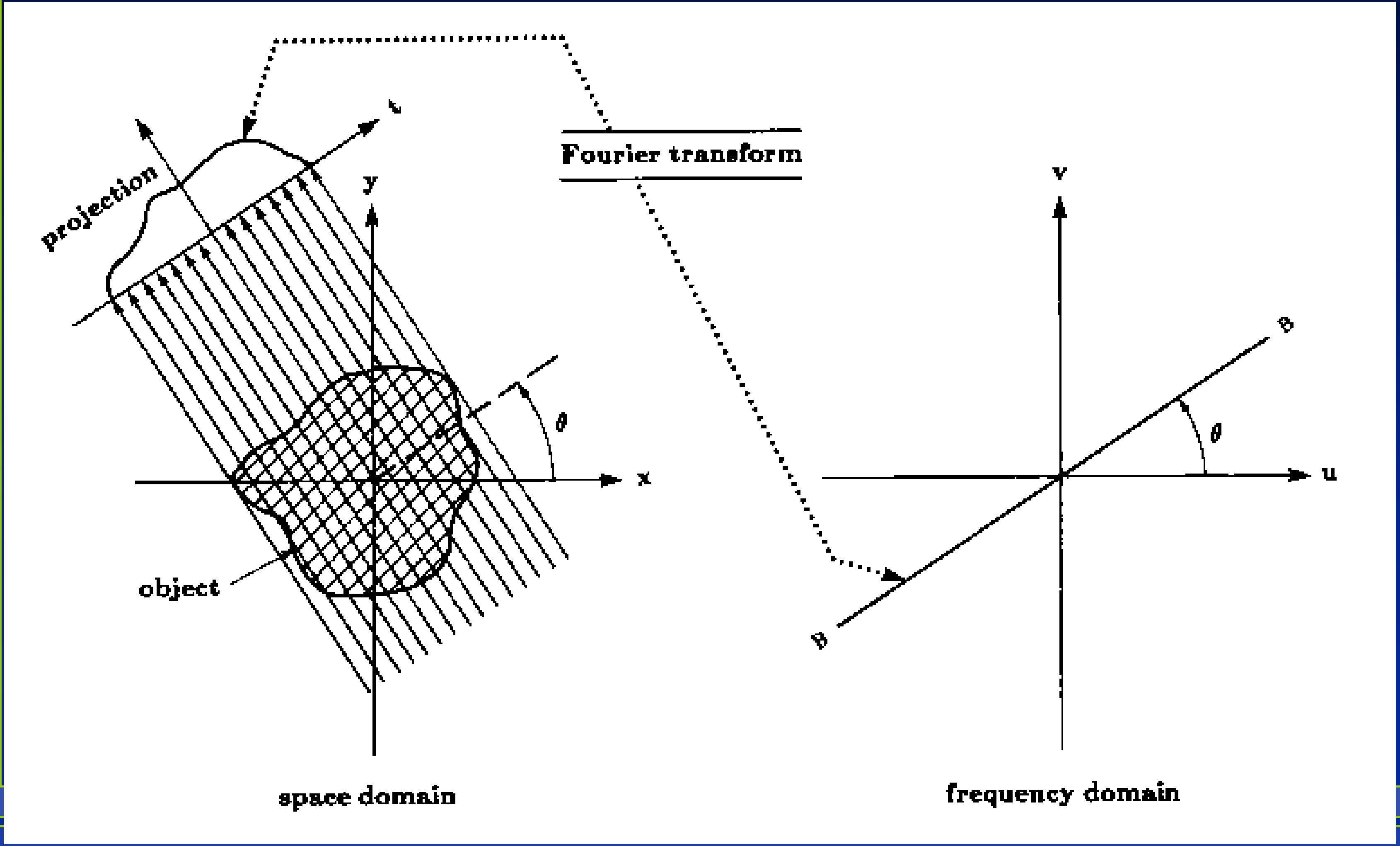
$f^{p}(x,y)$ is predicted pixel value

$w(x,y)$ is a weight corresponding to area of pixel within the width of each projection line pr

$N_{\theta}(\ell)$ is the sum of $w(x,y)$ along each pr

Fourier Slice Theorem:

1D Fourier transform of a parallel projection is equal to a slice of 2D FT of the original object



Fourier Slice Theorem:

FT of $pr_\theta(\ell)$

$$S_\theta(\omega) = \int_{-\infty}^{\infty} pr_\theta(\ell) e^{-j2\pi\omega\ell}$$

2D FT of object:

$$F(u,v) = \iint f(x,y) e^{-j2\pi(ux+vy)} dxdy$$

Fourier Slice Theorem:

1D FT of object along line v=0 or θ=0

$$F(u,0) = \iint \underbrace{f(x,y)}_{pr_{\theta=0}(l)} e^{-j2\pi ux} dx \underbrace{dy}$$

$$F(u,0) = \int pr_{\theta=0}(x) e^{-j2\pi ux} dx = S_{\theta=0}(u)$$

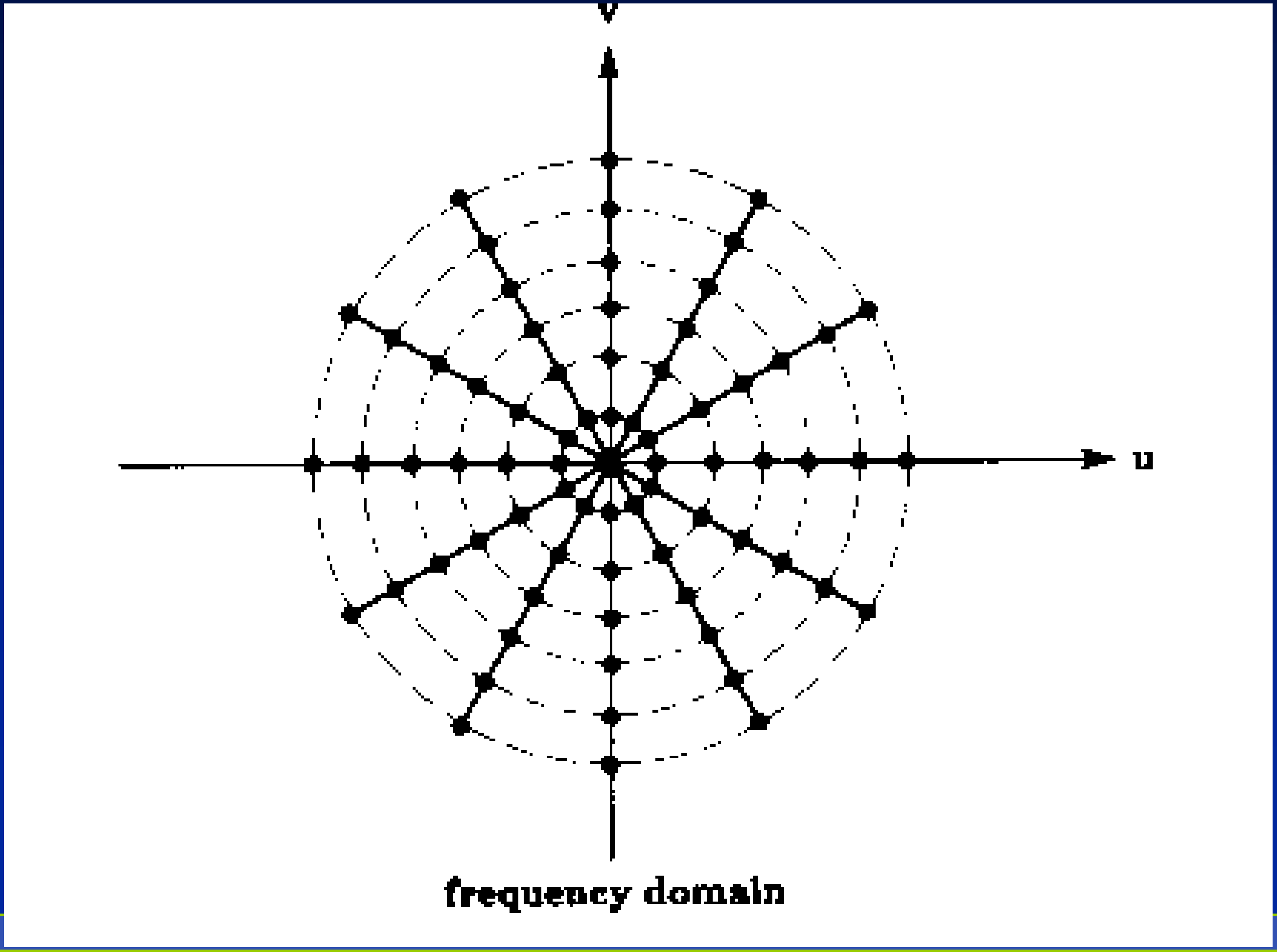
FT of $pr_\theta(\ell)$ in (l , s) coordinate system:

$$S_\theta(w) = \int \underbrace{\int f(l,s)\,ds}\, e^{-j2\pi wl}\,dl =$$

$$\iint f(x,y)e^{-j2\pi w(x\cos\theta + y\sin\theta)}\,dxdy =$$

$$F(w\cos\theta, w\sin\theta)$$

Summing 1D FT of projections of object at a number of angles gives an estimate of 2D FT of the object (projections are inserted along radial lines)



$2\pi|\omega|/K$ (K=180)

$\omega = \sqrt{(u^2=v^2)}$

Deconvolution filter

Algorithm for FT reconstruction:

1) For each angle θ between 0 to 180 for all (l)

2) Measure projection pr_θ

3) Fourier transform it to find S_θ

4) Multiply it by weighting function $2\pi|w|/K$ (K=180)
(ie. filtering (weighting) each FT data lines to estimate a pie-shaped wedge from line)

5- Inserting data from all projections into 2D FT place

6- Doing interpolation in frequency domain to fill the gap in high frequency regions.

5) Inverse FT
6) Interpolation data if necessary

Continuous and discrete version of the Shepp and Logan filter to reduce the emphasis given by HF components