Point-Source Geometry

Photon density at $I_{detector}$ due to Source obliquity

Assumption:

No resolution loss yet due to source.

Each ray independent of neighbors

For Parallel Rays: $I_d\,(x,y) = I_0\, e^{-\int \mu\,(x,y,z)\, dz}$

<u>Limitations</u>

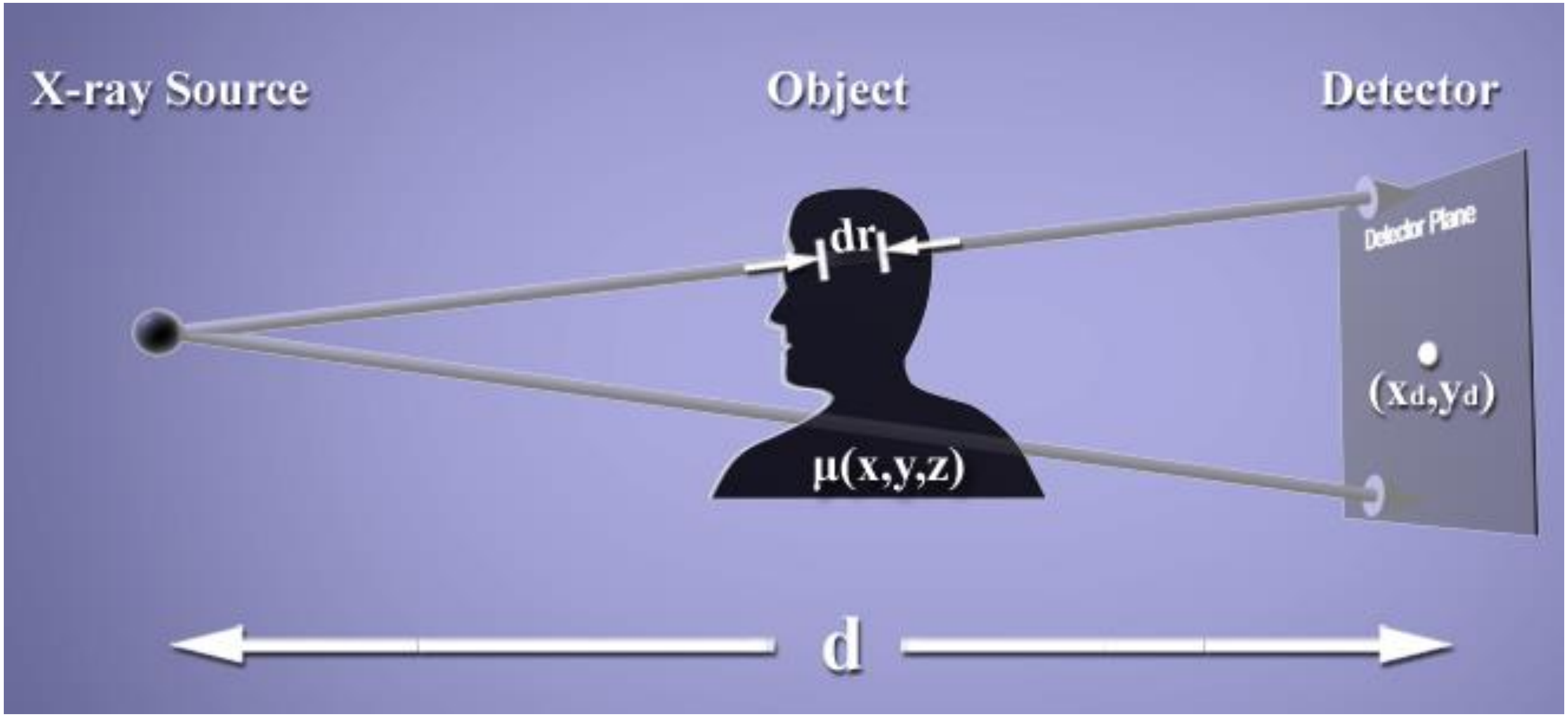
1)Finite Source produces rays that aren't parallel

Rays originate from a point source

2) Finite Detector

3) Distortion due to point source geometry

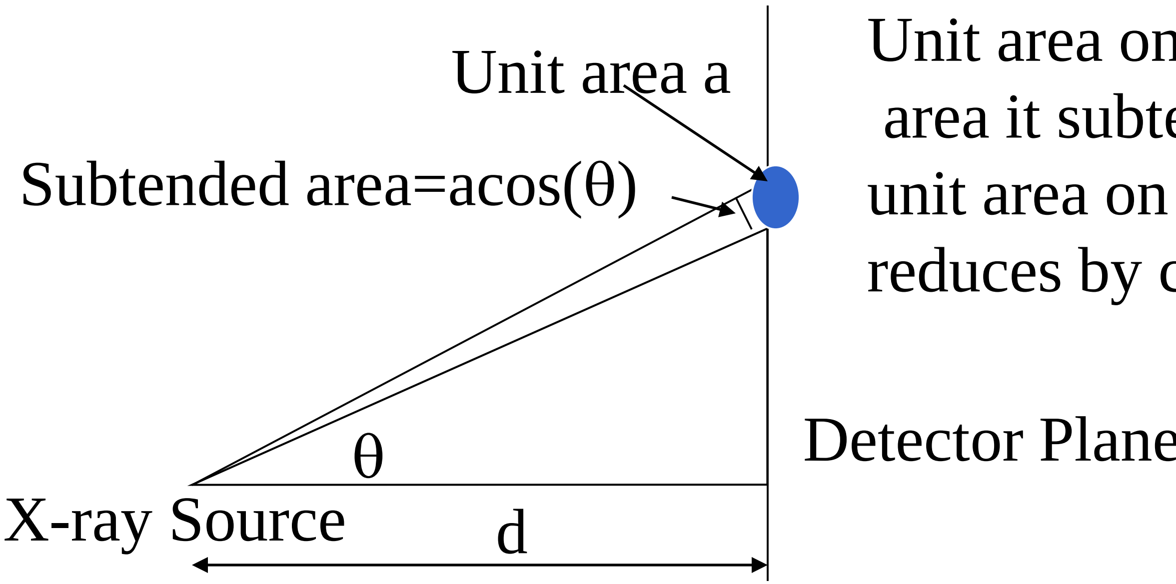
4) Resolution loss due to finite source size (not a point)



$$I_d\ (x_d,y_d) = I_i\ (x_d,y_d)\ \exp\ [\ -\int \mu_0\ (x,y,z)\ dr$$

Photon density at $I_{detector}$

First, what if no object?



Unit area on sphere is smaller than area it subtends on detector. So for unit area on detector, area on sphere reduces by cos(θ).

For small solid angles $\Omega \approx$ area/distance^2 = a (cos θ)/r^2

$\frac{\Omega}{4\pi}$ is fraction of radiation from the source covering a full sphere subtends 4π steradians, intercepted by Detector

$$I_d = KN\frac{\frac{\Omega}{4\pi}}{a}$$

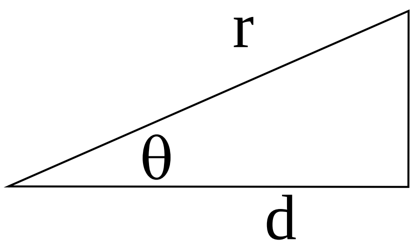
N photons emitted by source
K energy per photon
Divide by area a to normalize to detector area

$$I_d = KN\frac{\cos(\theta)}{4\pi r^2}$$

← Obliquity
← Inverse Square Law

Normalize to $I_d(0,0) = I_0$

$$I_0 = \frac{KN}{4\pi d^2}$$



$$r_d = \sqrt{x_d^2 + y_d^2}$$

Now rewriting I_i in terms of I_0 gives,

$$I_d = I_o \frac{d^2}{r^2} \cos(\theta)$$

$$I_d = I_o e^{-\tau\left(\frac{x_d}{M}, \frac{y_d}{M}\right)}$$

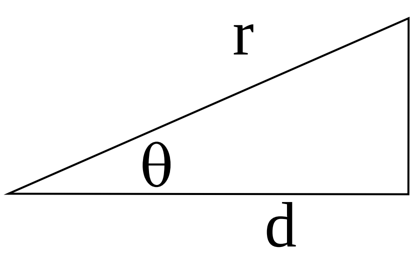
But $\dfrac{d}{r} = \cos(\theta)$

So,

$I_d = I_o \cos^3(\theta)$

Source Intensity on Detector Coordinates

$$I_0 = \frac{KN}{4\pi d^2}$$



$$r_d = \sqrt{x_d^2 + y_d^2}$$

$$I_d = I_o \cos^3 \theta$$

$$\cos(\theta) = \frac{d}{\sqrt{d^2 + r_d^2}} = \frac{1}{\sqrt{1 + \frac{r_d^2}{d^2}}}$$

$$I_d = I_0 \frac{1}{(1 + \frac{r_d^2}{d^2})^{3/2}}$$

$\cos(\theta)$ - Obliquity

$\cos^2 \theta$ - Inverse Square Law

Practical Example

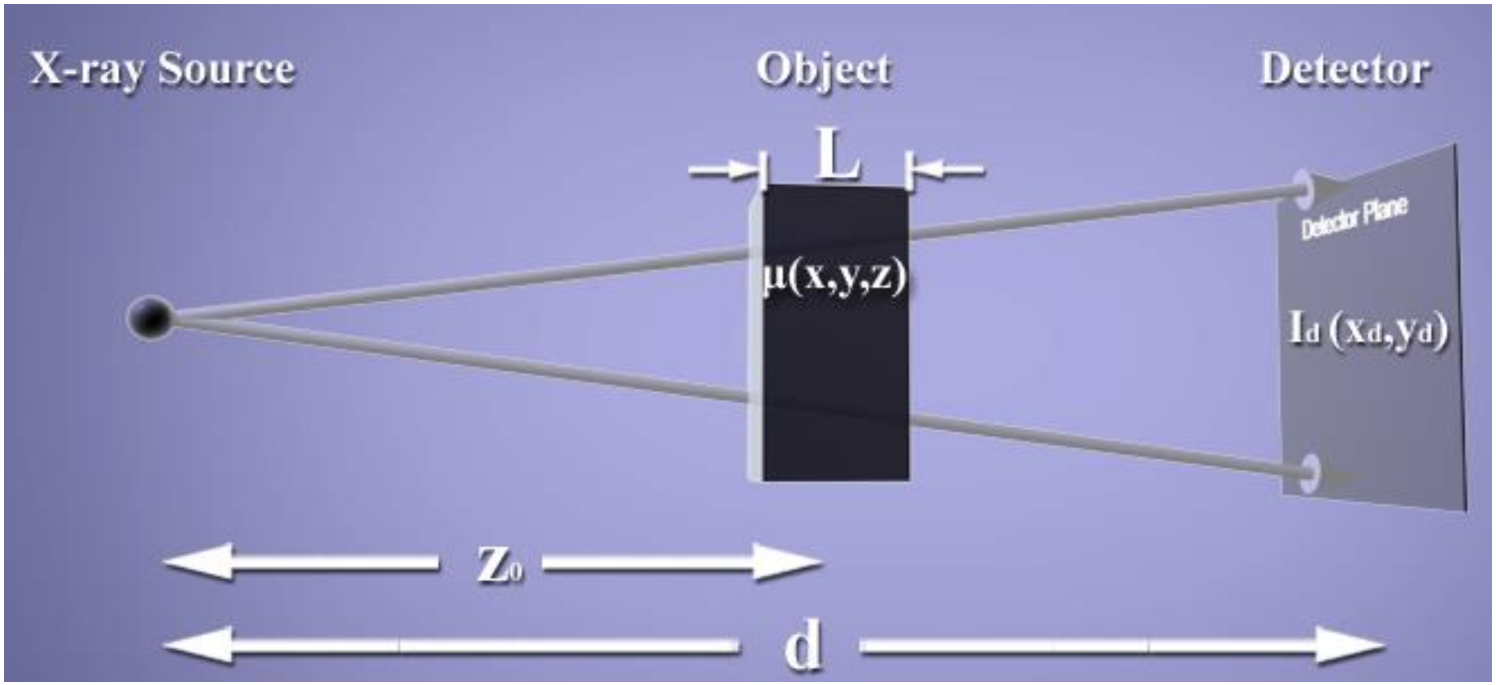
- For a 40 cm FOV with x-ray source 1 m away, how much amplitude modulation will we have due to source obliquity?

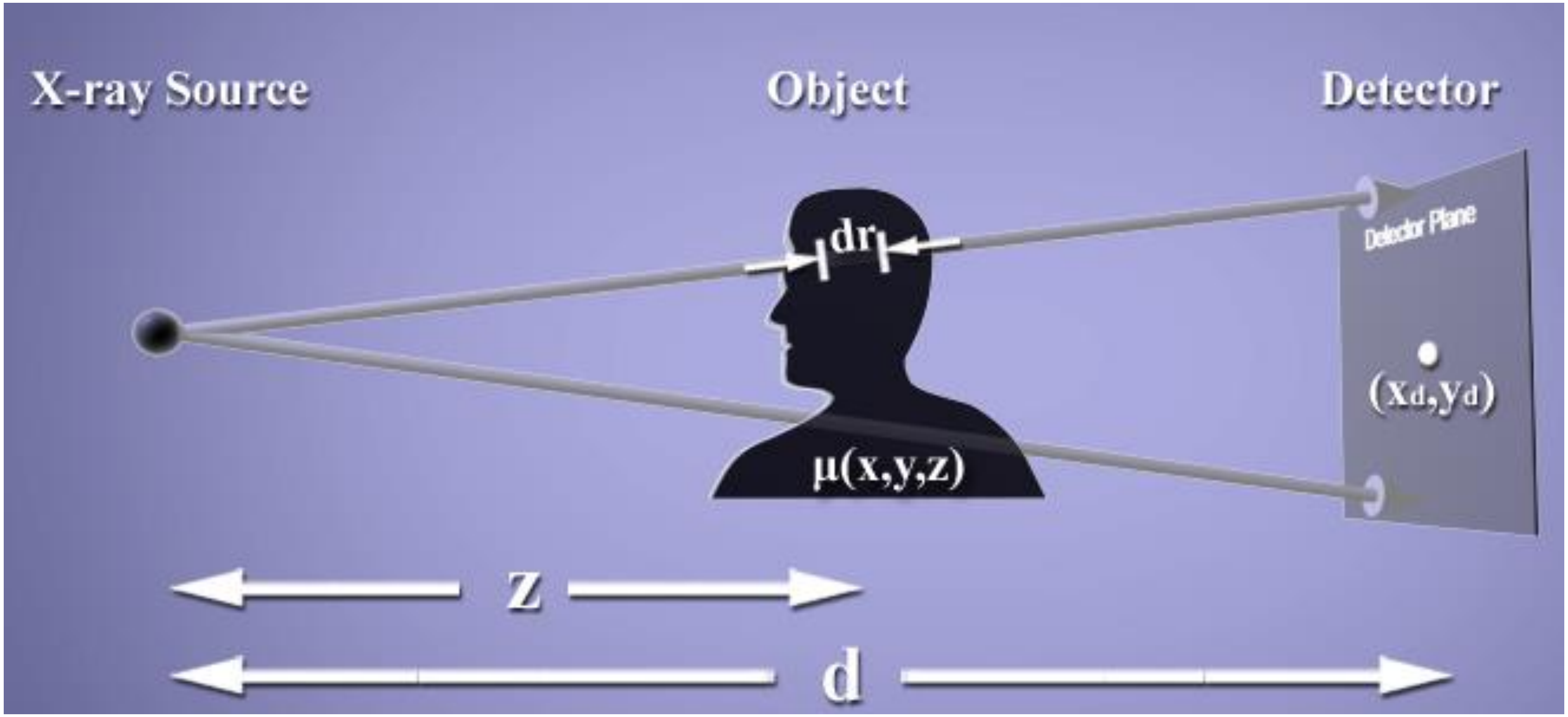
$$I_d = I_0 \frac{1}{(1+\frac{r_d^2}{d^2})^{3/2}}$$

Depth Dependent Magnification

Depth Dependent Magnification

Photon density change due to object obliquity





An incremental path of the x-ray, dr, can be described by its x, y, and z components.

$$dr = \sqrt{dx^2 + dy^2 + dz^2}$$

$$\frac{y_d}{y} = \frac{d}{z} \qquad \frac{x_d}{x} = \frac{d}{z}$$

Each point in the body (x,y) can be defined in terms of the detector coordinates it will be imaged at.

$$x = \frac{x_d}{d} z \qquad y = \frac{y_d}{d} z$$

Let's make intensity expression parametric in z.

$$dr = dz\sqrt{1+(dx/dz)^2+(dy/dz)^2} \qquad dr = dz\sqrt{1+(x_d/d)^2+(y_d/d)^2}$$

$$r_d = \sqrt{x_d^2+y_d^2} \quad \Longrightarrow \quad dr = dz\sqrt{1+r_d^2/d^2}$$

Then, rewriting an earlier description in terms of the detector plane. z/d describes minification to get from detector plane to object.

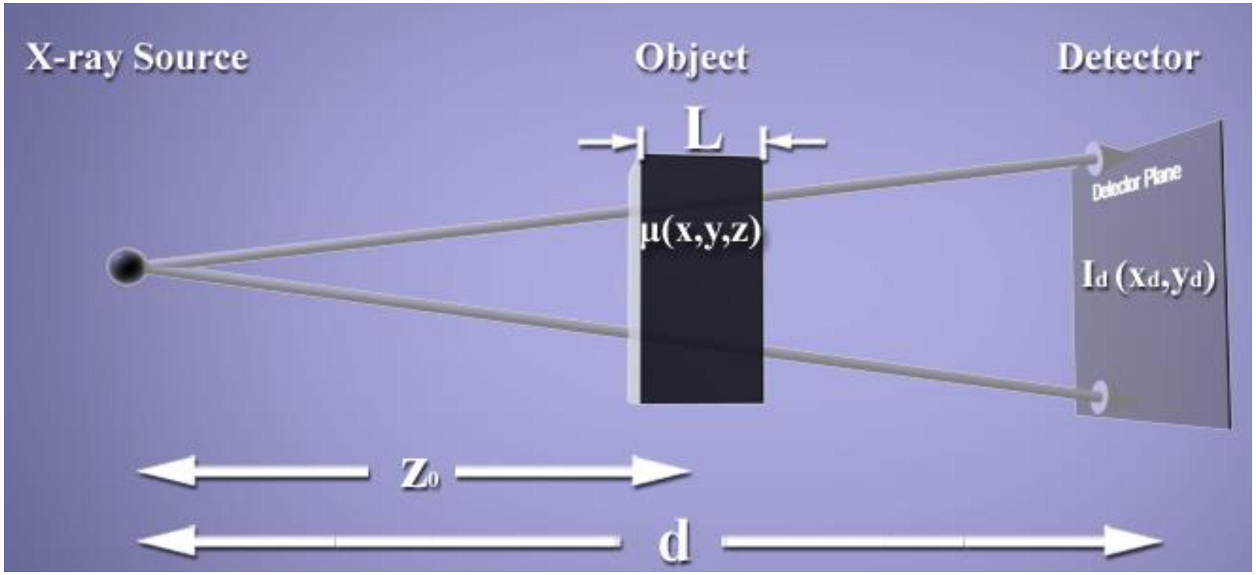
$$I_d\,(x_d,y_d) = I_i \exp\left[-\sqrt{1+\frac{r_d^2}{d^2}} \int \mu_o\,((x_d/d)z,\,(y_d/d)z,\,z)\,dz\right]$$

Putting it all together gives,

$$I_d = I_o / (((1 + r_d^2)/d^2)^{3/2}) \exp \left[- \sqrt{1 + \frac{r_d^2}{d^2}} \int \mu_o ((x_d/d)z, (y_d/d)z, z)\, dz \right]$$

source obliquity   object obliquity

Examples 1



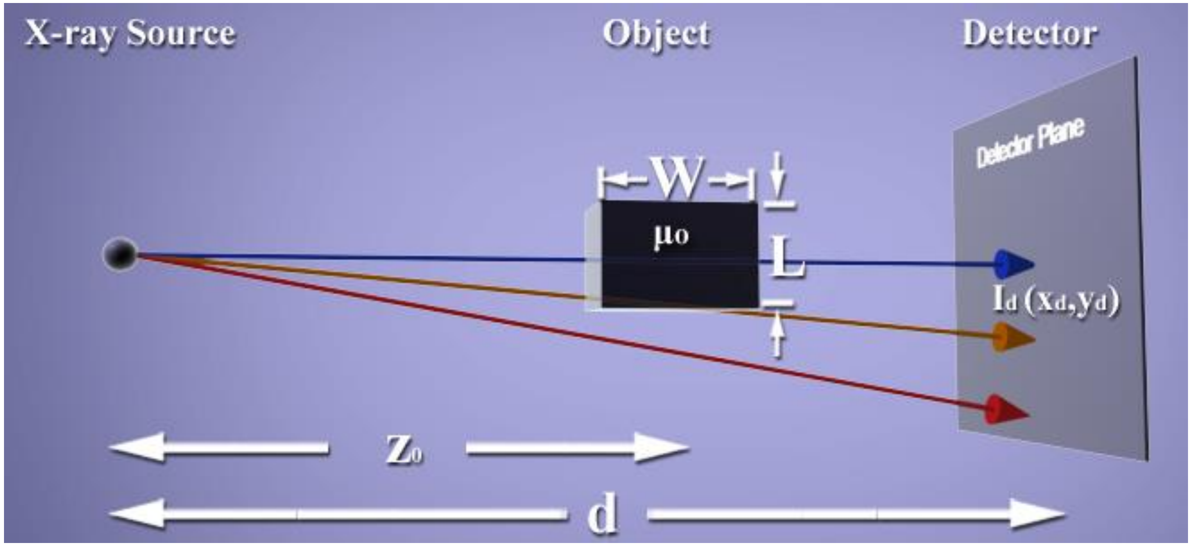
Object $\mu(x,y,z) = \mu_o \, \mathrm{rect}((z - z_o)/L)$
Object is not a function of x or y, just z.

$$I_d(x_d, y_d) = I_i \exp\left[-\sqrt{1+\frac{r_d^2}{d^2}}\; \mu_o L \right]$$

If we assume detector is entirely in the near axis, $r_d^2 << d^2$
Then, simplification results,

$$I_d = I_o \, e^{-\mu_0 L}$$

Example 2



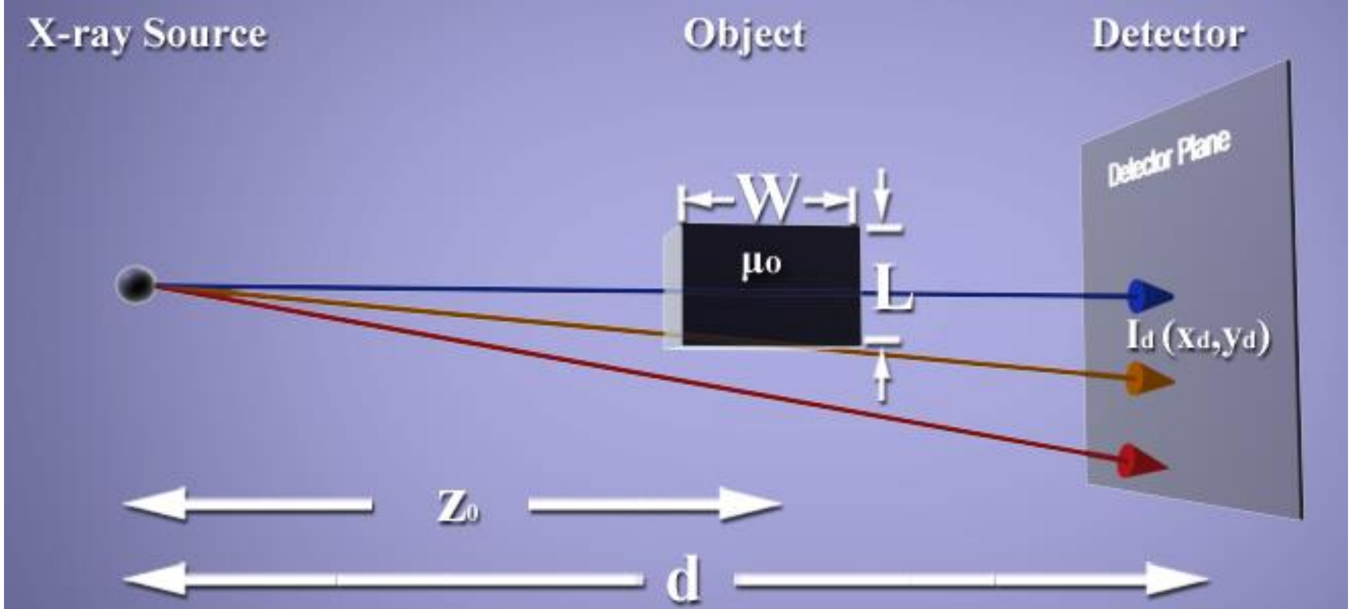
x out of plan
Infinite in x

For $\mu_o(x,y,z) = \mu_o \Pi(y/L) \Pi((z - z_0)/w)$
Find the intensity on the detector plane
Three cases:

1. Blue Line: X-Ray goes through entire object
2. Red Line: X-ray misses object completely
3. Orange Line: X-ray partially goes through object

$$I_d(x_d,y_d) = I_i \exp\left[- \sqrt{1+\frac{r_d^2}{d^2}} \int \mu_o((x_d/d)z, (y_d/d)z, z)\, dz\right]$$

Example 2



$$I_d\,(x_d,y_d) = I_i \exp\left[-\sqrt{1+\frac{r_d^2}{d^2}}\int \mu_o\,((x_d/d)z,\,(y_d/d)z,\,z)\,dz\right]$$

For the blue line, we don't have to worry that the path length through the object will increase as r_d increases. That is taken care of by the obliquity term $\sqrt{1+\frac{r_d^2}{d^2}}$

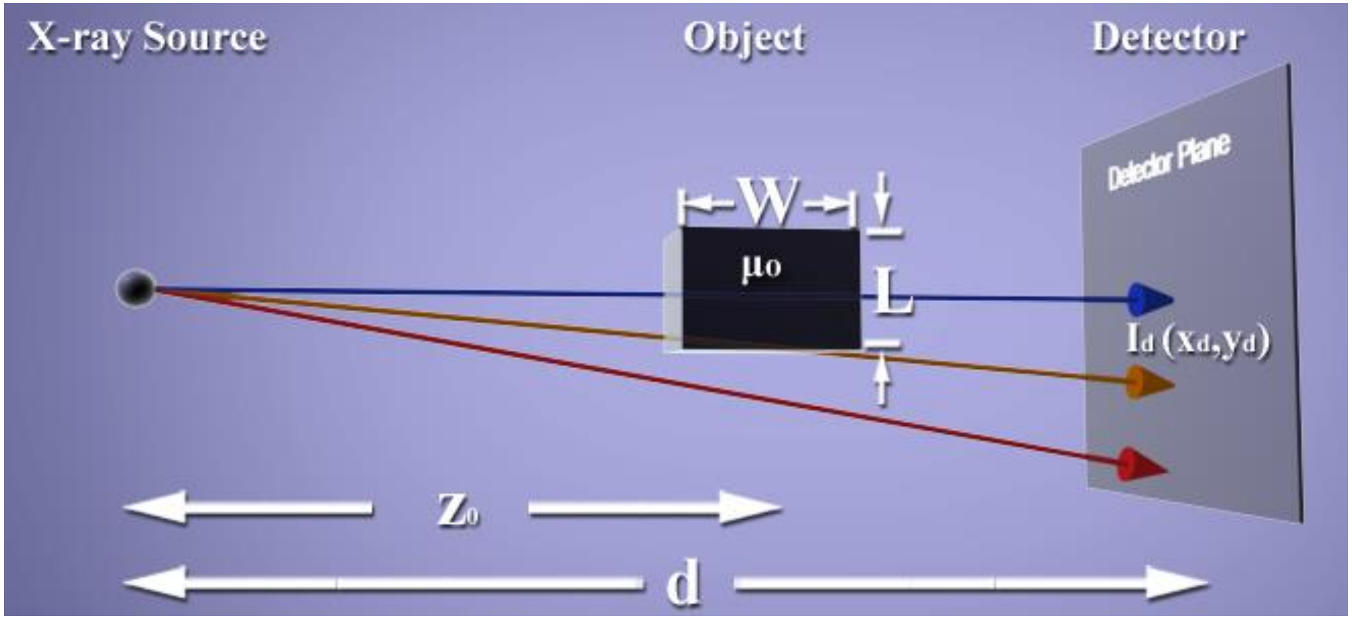
For the red path, $I_d\,(x_d,y_d) = I_i$

For the orange path, the obliquity term will still help describe the lengthened path.

But we need to know the limits in z to integrate

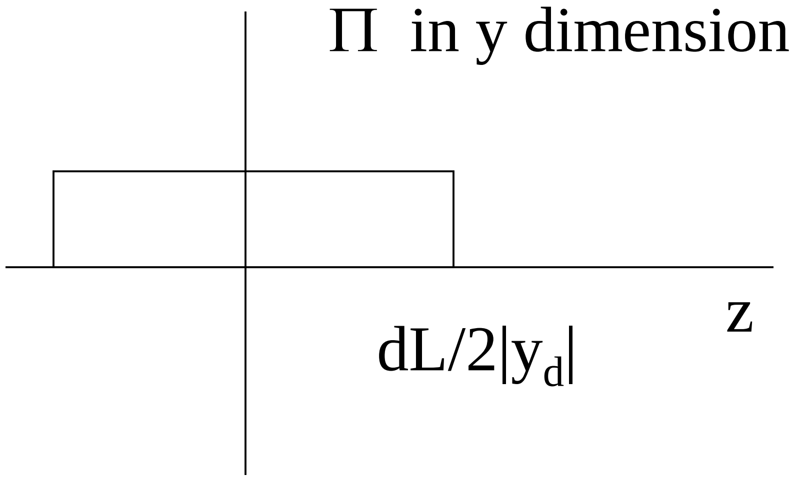
$\mu_o (x,y,z) = \mu_o \Pi (y/L) \Pi((z - z_0)/w)$

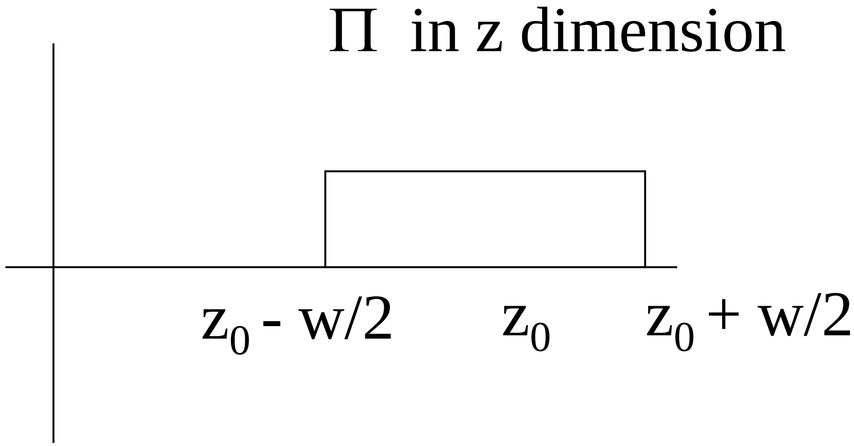
Example 2



If we think of thin planes along z, each plane will form a rect in y_d of width dL/z. Instead of seeing this as a Π in y, let's mathematically consider it as a Π in z that varies in width according to the detector coordinate y_d. Then we have integration only in the variable z. The Π define limits of integration

$$I_d(x_d, y_d) = I_i \exp[-\sqrt{1+\frac{r_d^2}{d^2}}\mu_0 \int \Pi(\frac{z}{\frac{dL}{y_d}})\Pi(\frac{z-z_0}{w})dz$$





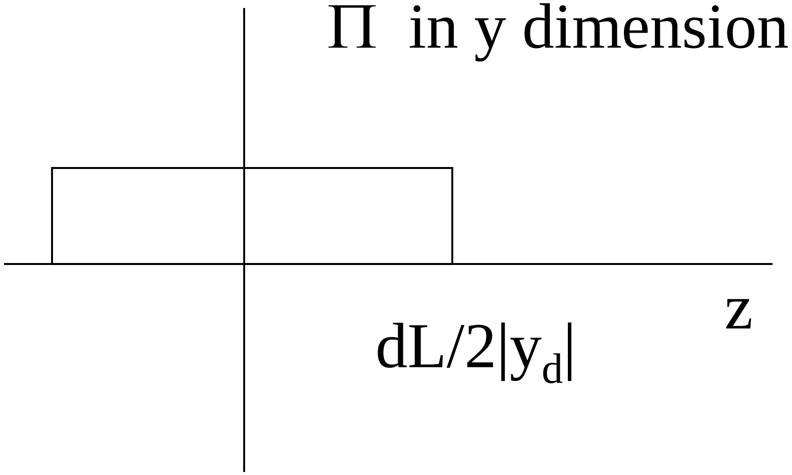
1) <u>X-ray misses object completely</u>

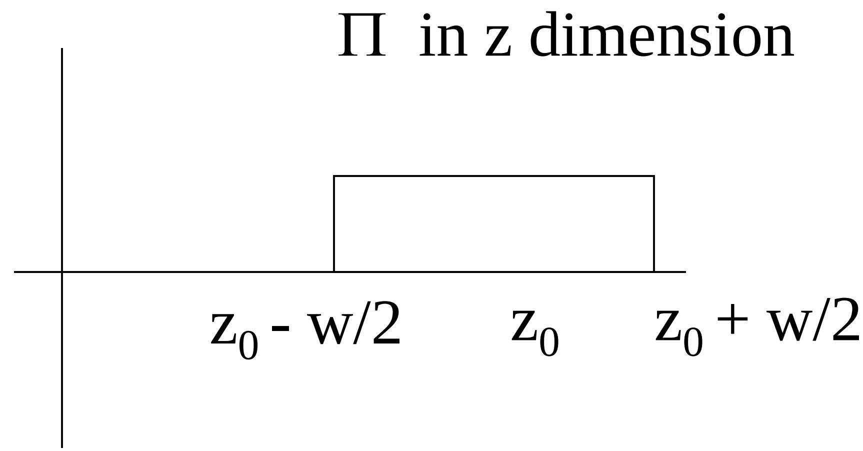
As y_d grows,first Π contracts and no overlap exists between the Π ,
functions. No overlap case when

$dL/2y_d < z_0 - w/2$

$|y_d| > dL/(2 z_o - w)$

$I_d = I_i$





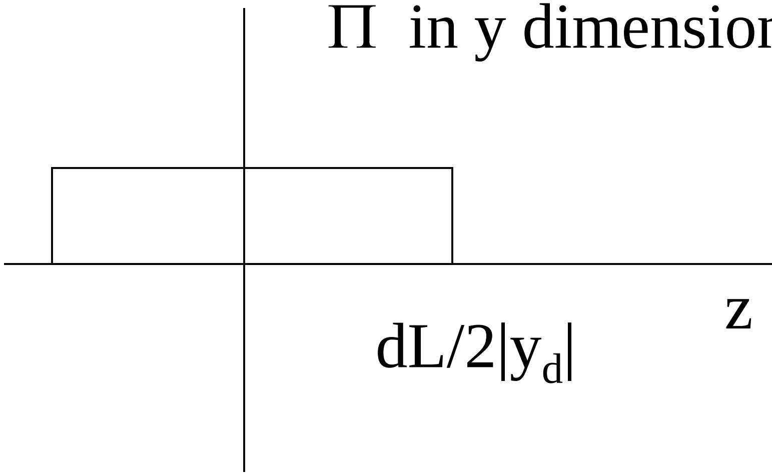
1) <u>X-ray goes completely through object</u>

As $y_d \rightarrow 0$, x-ray goes completely through object
This is true for $dL/2y_d > z_0 + w/2$

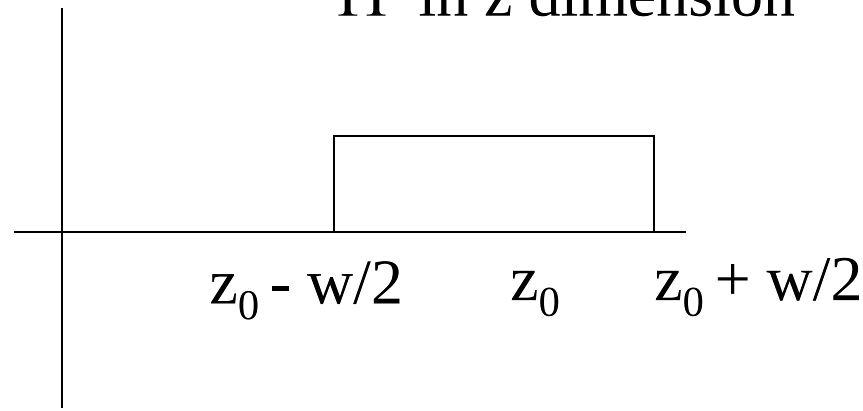
$$|y_d| < dL/(2 z_o + w)$$

$$I_d = I_i \exp\{-\mu_o \sqrt{1+\frac{r_d^2}{d^2}}\, w\}$$

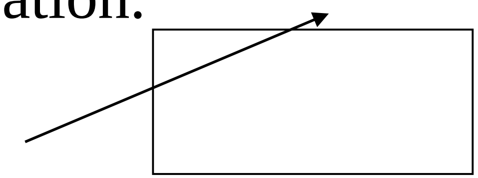
Π in y dimension



Π in z dimension



3) Partial overlap case. Picture above gives the limits of integration.



$$I_d = I_i \exp\left\{-\mu_o \int_{\min\left\{zo - w/2,\; dL/2|y_d|\right\}}^{\min\left\{zo + w/2,\; dL/2|y_d|\right\}} - \sqrt{1+\frac{r_d^2}{d^2}}\; dz\right\}$$

3) $I_d =$

$$I_i \exp\{-\mu_o \sqrt{1+\frac{r_d^2}{d^2}} \; ((dL/ 2|y_d|) + w/2 - z_o)\}$$

X-rays miss object

X-rays partially travel through object

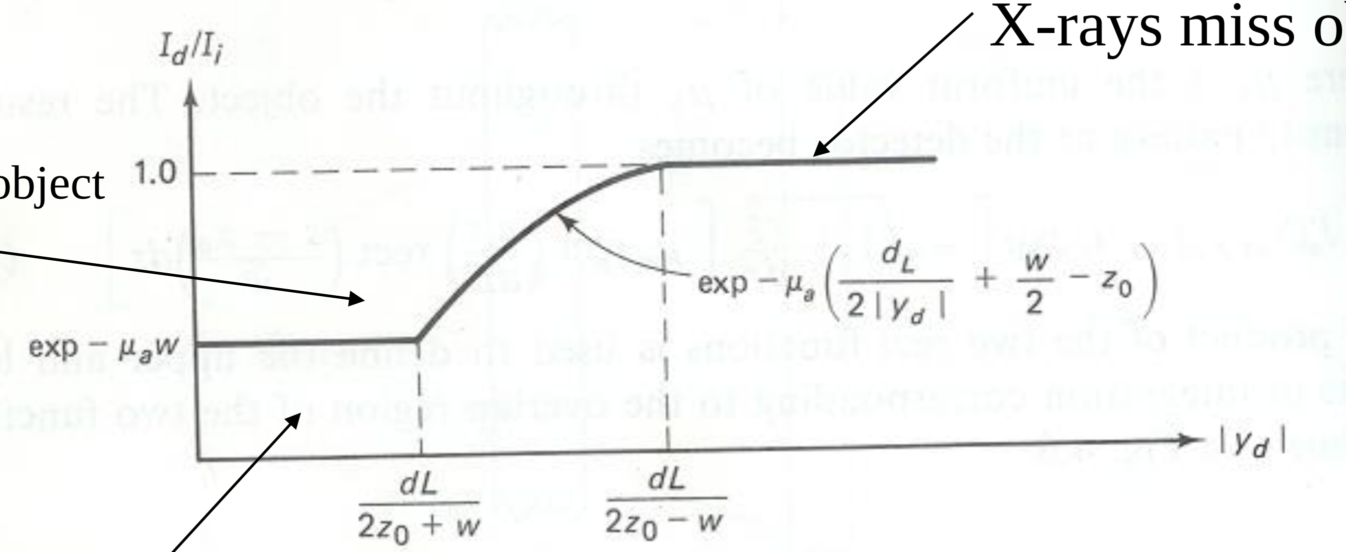


FIG. 4.7 Transmission of a rectangular object.

X-rays go entirely through object

The above diagram ignores effects of source obliquity and the factor $\sqrt{1+\frac{r_d^2}{d^2}}$ in the exponential How would curve look differently if we accounted for both of these?

Thin section Analysis

See object as an array of planes

$\mu\ (x,y,z) = \tau\ (x,y)\ \delta\ (z - z_0)$

Analysis simplifies since only one z plane

$$I_d = I_i e^{\left(-\sqrt{1+\frac{r_d^2}{d^2}}\,\tau\left(\frac{x_d}{M},\frac{y_d}{M}\right)\right)}$$

where $M = d/z_0$ represents object magnification

If we ignore obliquity,

$$I_d = I_o e^{\left(-\tau\left(\frac{x_d}{M},\frac{y_d}{M}\right)\right)}$$

Or in terms of the notation for transmission, t,

$$I_d = I_o t\left(\frac{x_d}{M},\frac{y_d}{M}\right)$$

Note: No resolution loss yet. Point remains a point.