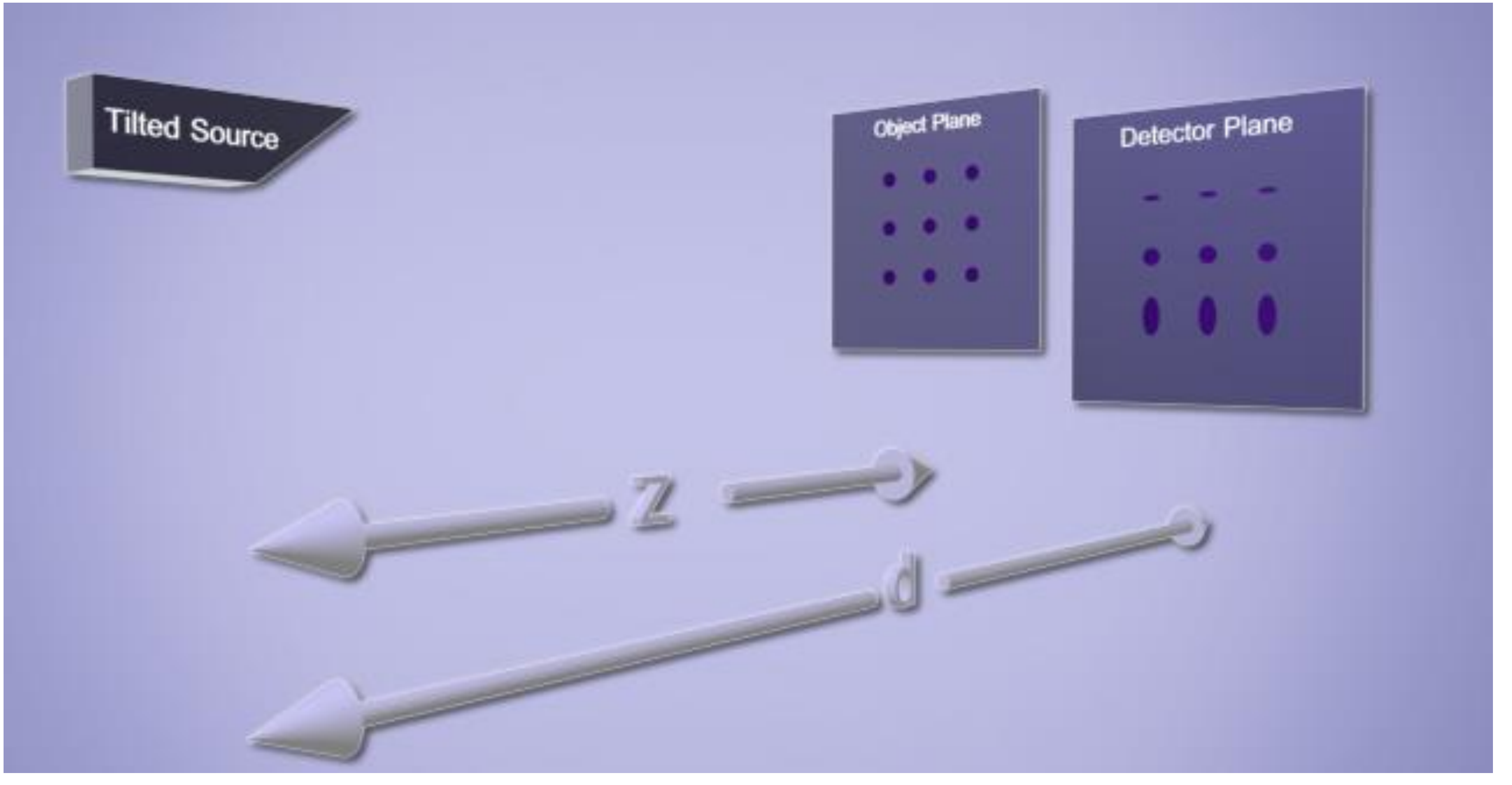
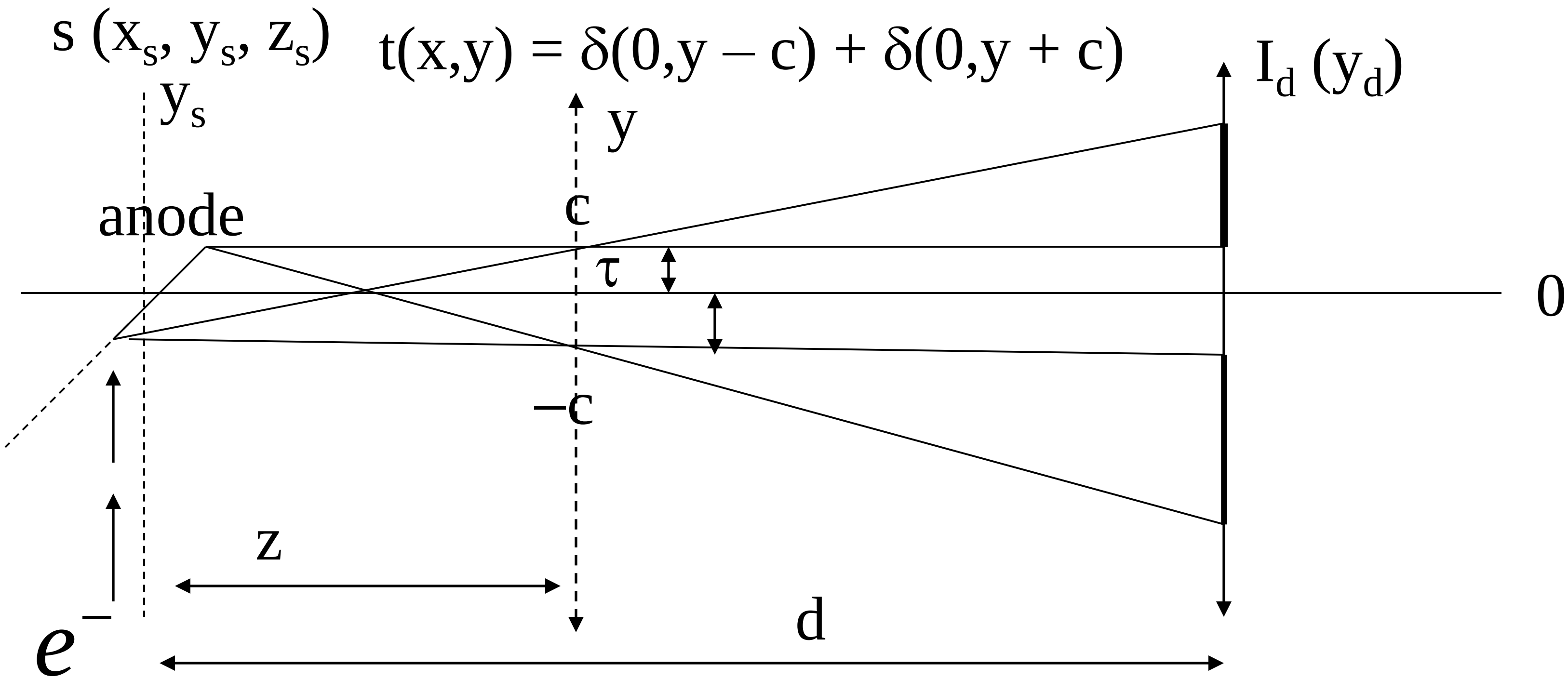
Effects of Tilted Source (Anode Angle)

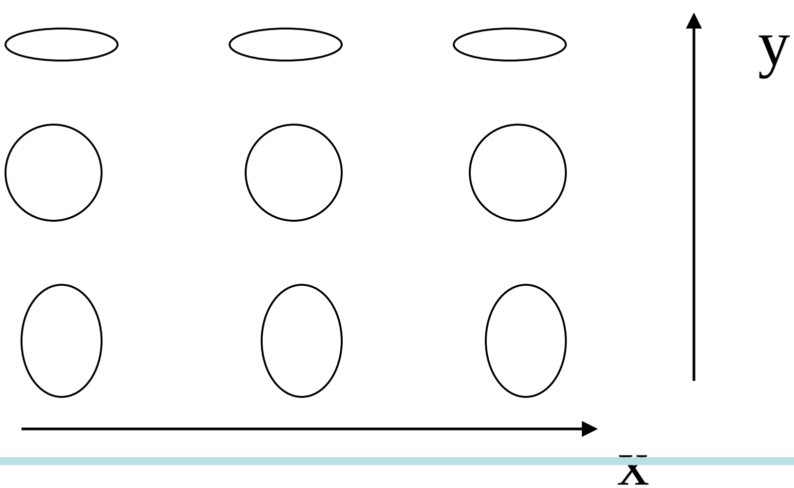


Source magnification in x_d function of z
Source magnification in y dependent on y_d and z

We will skip the mathematical development on this section



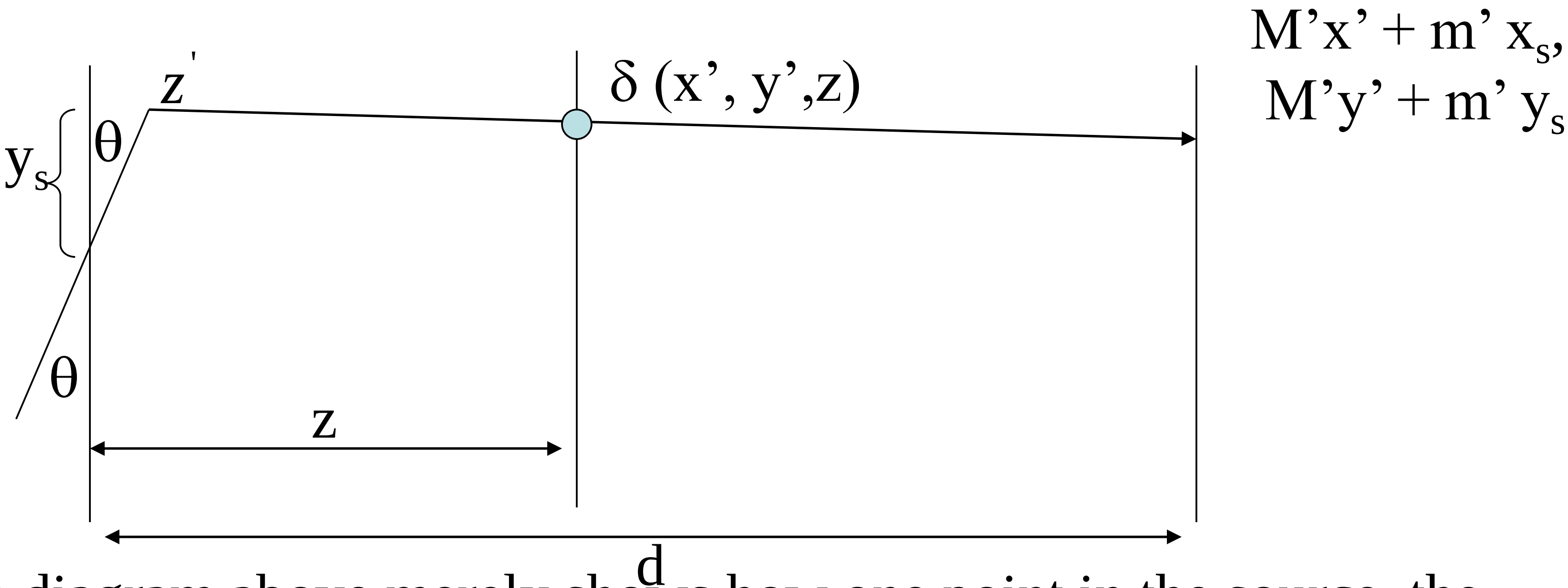
Consider a 3 x 3 array of pinholes



t(x,y) above can be considered a lead plate with two pinholes punched into it.

3 x 3 array of circular pinholes to left shows how source is contracted in y for positive y detector positions and enlarged for negative detector locations.

Let’s allow the magnification to be different on each axis.
The pinhole models an object, $\delta(x', y', z)$.



The diagram above merely shows how one point in the source, the pinhole, and the detector are related by geometry.

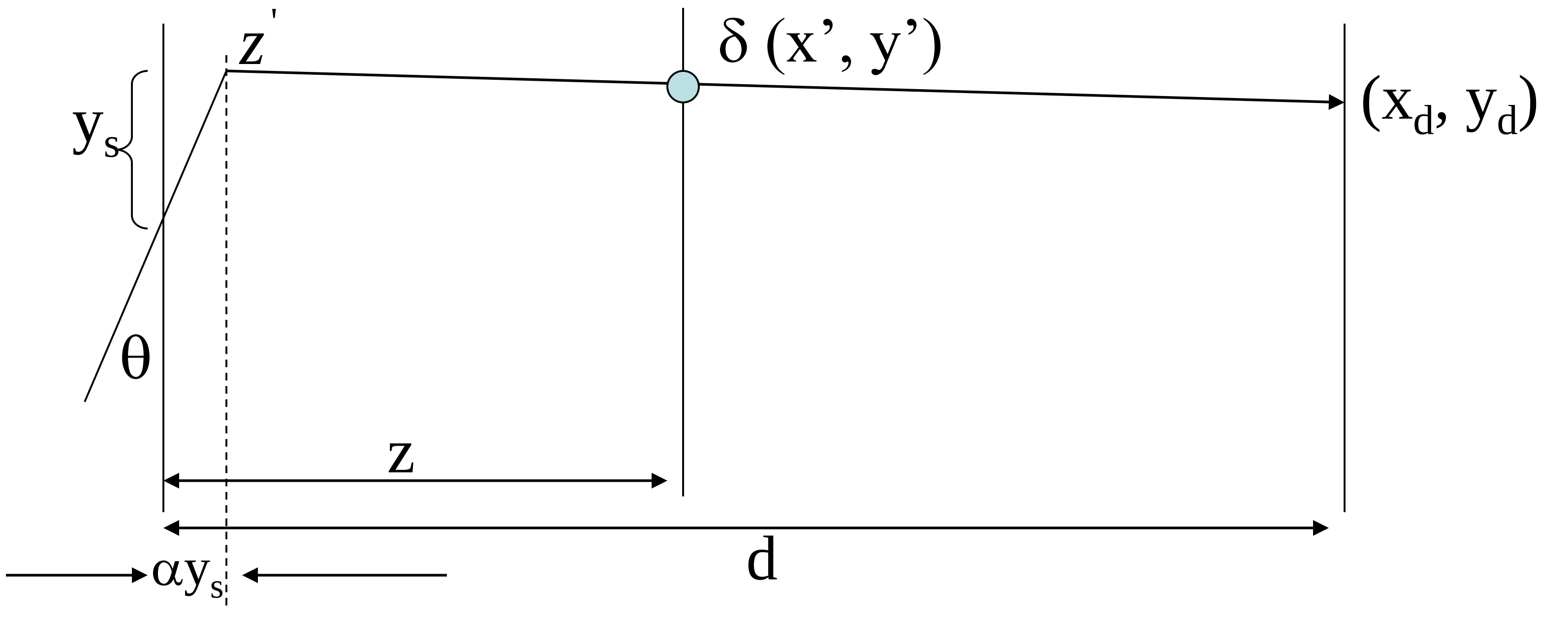
Here we use a M’ to allow for magnification of the pinhole, and m’ to allow for magnification of the source

We will consider the impulse response using a pinhole at x', y'.

$$h(x_d, y_d, x', y') = \frac{\eta}{m_x m_y} s(\frac{x_d - M_x x'}{m_x}, \frac{y_d - M_y y'}{m_y})$$

where η is a collection efficiency for the pinhole: $\eta = \Omega / 4\pi d^2$

If $\tan(\theta) = \alpha$,
then the source position $z' = \alpha\, y_s$



By geometry, object magnification is

$$M' = (d - \alpha\, y_s)/(z - \alpha\, y_s)$$

Source magnification is

$$m' = -\,((d - \alpha\, y_s) - (z - \alpha\, y_s)) / (z - \alpha\, y_s)$$

$$m' = -\,(d - z)/ (z - \alpha\, y_s)$$

Recall $x_d = M'x' + m'x_s$

$y_d = M'y' + m' y_s$

$$M_x = \partial x_d / \partial x' = M' = (d - \alpha\, y_s)/(z - \alpha\, y_s) \approx d/z$$

since for practical arrangements d, z >> αy_s

Typical dimensions: z, d ~ 1 m, y_s ~ 1mm

Similarly $M_y = \partial y_d / \partial y' \approx d/z$

$m_x = \partial x_d / \partial x_s = m' \approx -(d-z)/z = m$

$m_y = \partial y_d / \partial y_s$

This is more interesting derivative since both M' and m' are functions of y_s

$m_y = \partial y_d / \partial y_s = \partial (M' y')/ \partial y_s + \partial (m' y_s)/ \partial y_s$

From previous slide,

$M' = (d - \alpha y_s)/(z - \alpha y_s)$ and $m' = -(d - z)/(z - \alpha y_s)$

To find : $m_y = \partial y_d / \partial y_s = \partial(M' y')/\partial y_s + \partial(m' y_s)/\partial y_s$

$$m_y = [((z - \alpha y_s)(-\alpha) - (d - \alpha y_s)(-\alpha)]\, y')/(z - \alpha y_s)^2$$
$$+ -(d-z) \bullet [(z - \alpha y_s) - y_s(-\alpha)]/(z - \alpha y_s)^2$$
$$= (-\alpha[z-d]\, y' - (d-z)\, z)/(z - \alpha y_s)^2 = -(d-z)(z - \alpha y')/((z - \alpha y_s)^2$$

$m_y = -(d-z)(z - \alpha y')/z^2 = m\,(1 - (\alpha y'/z))$

Using this relationship and ignoring obliquity,

$$h(x_d, y_d, x', y') = \frac{1}{4\pi d^2 m^2} \frac{1}{(1 - \alpha y'/z)} s\left(\frac{x_d - M x'}{m}, \frac{y_d - M y'}{m(1 - \alpha y'/z)}\right)$$

How does magnification change with object position?

Since system is linear, we can write a superposition integral.

$I_d(x_d, y_d) = \int\int h((x_d, y_d, x', y') \, t(x',y') \, dx' \, dy' =$

$$\iint \frac{1}{4\pi d^2 m^2} \frac{1}{(1-\alpha y'/z)} s\left(\frac{x_d - M x'}{m}, \frac{y_d - M y'}{m(1-\alpha y'/z)}\right) t(x', y') dx' dy'$$

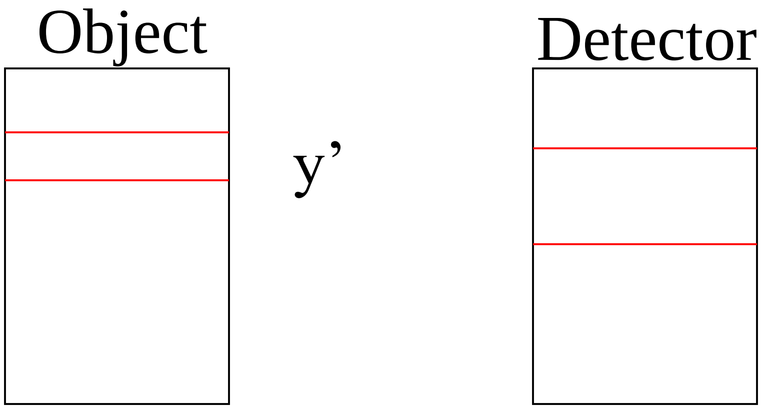
For developing space-invariance, let’s consider a magnified object

Let
$$x'' = Mx' \qquad y'' = My'$$
$$dx'' = Mdx' \qquad dy'' = Mdy'$$

$$I_d(x_d, y_d) = \frac{1}{4\pi d^2 m^2} \iint \frac{1}{(1-\alpha y''/Mz)} s\left(\frac{x_d - x''}{m}, \frac{y_d - y''}{m(1-\alpha y''/Mz)}\right) t\left(\frac{x''}{M}, \frac{y''}{M}\right) dx'' dy''$$

Not a space-invariant system since
1- (αy”/Mz) varies slowly with y” or y’

But it doesn’t vary much in a region of an object.



Let $y'_d = My'$

Consider a horizontal strip across the detector centered at

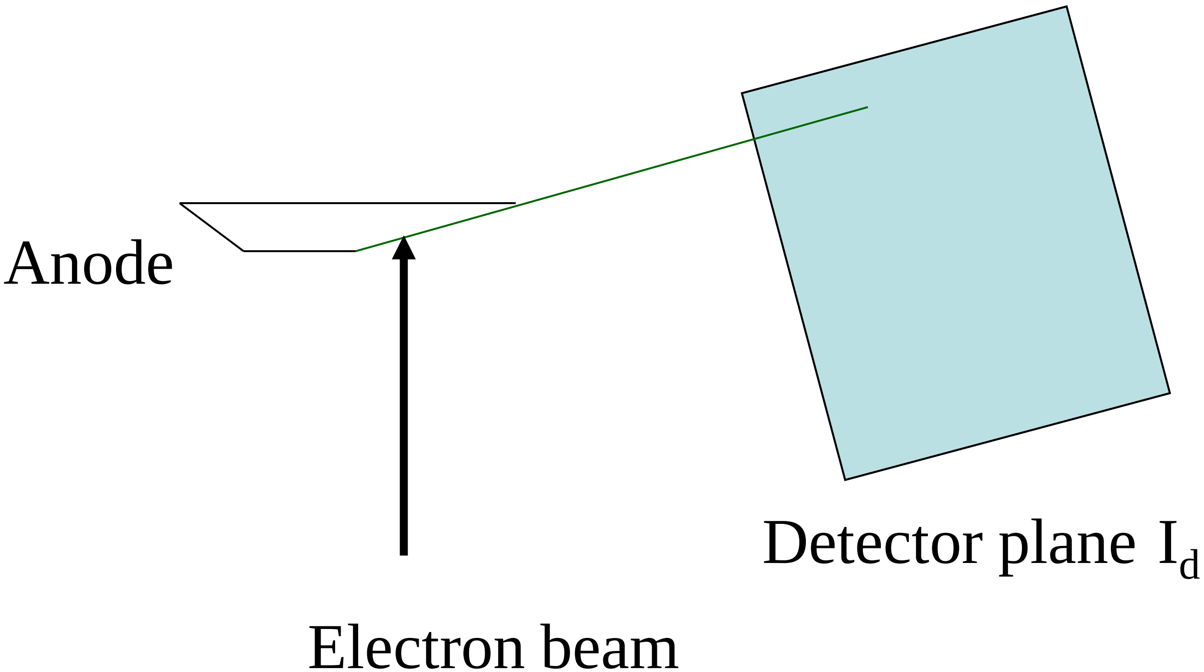
For this region, $\alpha y'' = \alpha M y' = \alpha y_d$

where $Mz = d$

$$I_d(x_d, y_d, y_{d'}) = \frac{1}{4\pi d^2 m^2 (1 - \alpha y_d' / d)} s\left(\frac{x_d}{m}, \frac{y_d}{m(1 - \alpha y_d' / d)}\right) ** t\left(\frac{x_d}{M}, \frac{y_d}{M}\right)$$

Here y_d' is a constant over a region in the detector during the convolution.

At $\alpha y' = z$, source width goes to 0 in y. We call this the "heel effect"



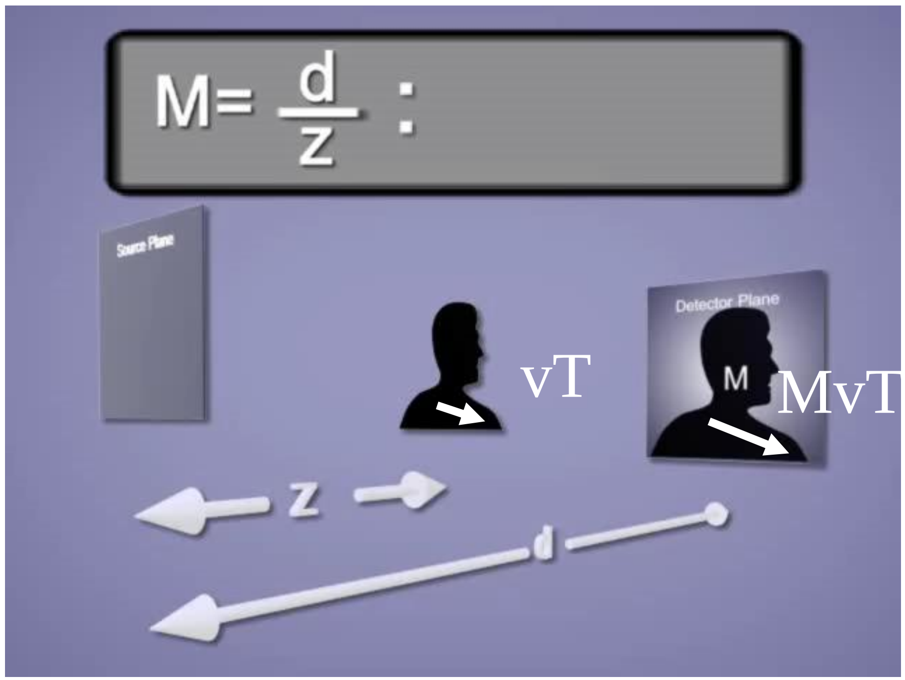
In the frequency domain,

$$I_d(u, v, y_{d'}) = \frac{M^2}{4\pi d^2} S\left(mu, m\left(1 - \frac{\alpha y_d'}{d}\right)v\right) T(Mu, Mv)$$

Effect of Motion

Effect of Motion

Let's consider object motion with constant velocity in the x direction over the imaging time T at velocity v



Over time T, the object size will change position in the detector plane by MvT

The impulse response due to movement in x is

$$h_{movement}(x_d, y_d) = \frac{1}{MvT}\Pi(\frac{x_d}{MvT})\delta(y_d)$$

Notice that there is no degradation in y, as we expected.

The complete impulse response is given below(a planar source parallel to the detector is used here for simplicity).

$$I_d = \frac{1}{4\pi d^2 m^2} t(\frac{x_d}{M}, \frac{y_d}{M}) ** s(\frac{x_d}{m}, \frac{y_d}{m}) * \frac{1}{MvT}\Pi(\frac{x_d}{MvT})$$

Blur in x direction gets minimized as T decreases to 0

How to minimize Motion blurring

Assume a L x L source parallel to detector

$$s(x_s, y_s) = K\Pi(\frac{x_s}{L})\Pi(\frac{y_s}{L})$$

Write energy density E_s as a power integrated over time

$p(x_s, y_s)$ is regional source power density (it is limited by tungsten melting point).

Set $p(x_s, y_s) = P_{max.}$ Operating tube at maximum power available.

T is the time beam is ON

$E_s = \iiint p(x_s, y_s)\, dx_s\, dy_s\, dt = P_{max} T L^2$, then $T = \dfrac{E_s}{P_{max} L^2}$

If L increases, source grows, then T can decreases.

Complete Response function for Rectangle Source and movement MvT in x direction:

$$h(x_d, y_d) = K\Pi(\frac{x_d}{mL}, \frac{y_d}{mL}) ** \Pi(\frac{x_d}{\frac{MvE_s}{P_{max} L^2}})$$

We have extension of the impulse response in the x direction as:

$$X = |m|L + (MvE_s/ P_{max}\, L^2)$$

by the convolution of two rectangles, due to source blurring and motion

We could choose to minimize several criteria. Area, for example. We will simply minimize X now with respect to L.

$$L_{min} = \left| \frac{2MvE_s}{P_{max}|m|} \right|^{1/3}$$

<u>Corresponding Exposure Time at Optimal L</u>

$$T = \frac{E_s}{P_{max} L^2} = \frac{E_s}{P_{max}} \left[\frac{P_{max}\,|\,m\,|}{2MvE_s} \right]^{2/3} = \left[\frac{E_s}{P_{max}} \right]^{1/3} \left[\frac{|\,m\,|}{2Mv} \right]^{2/3}$$

If v = 0, then L = 0 T = ∞ no source blurring

If |m| = 0, object is on detector L→ ∞ →T = 0