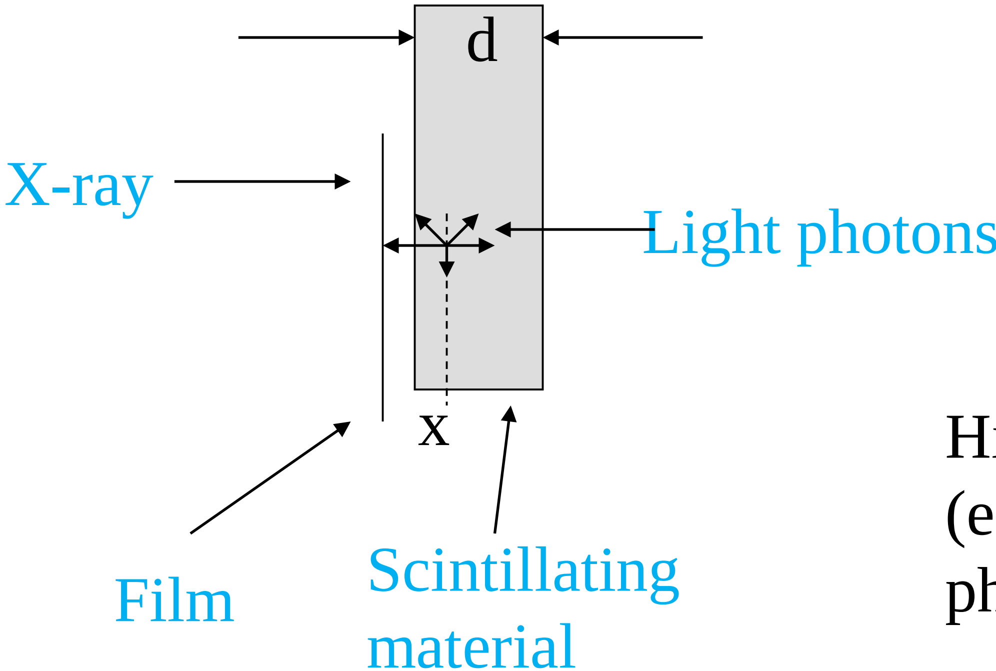
Screen Film Systems



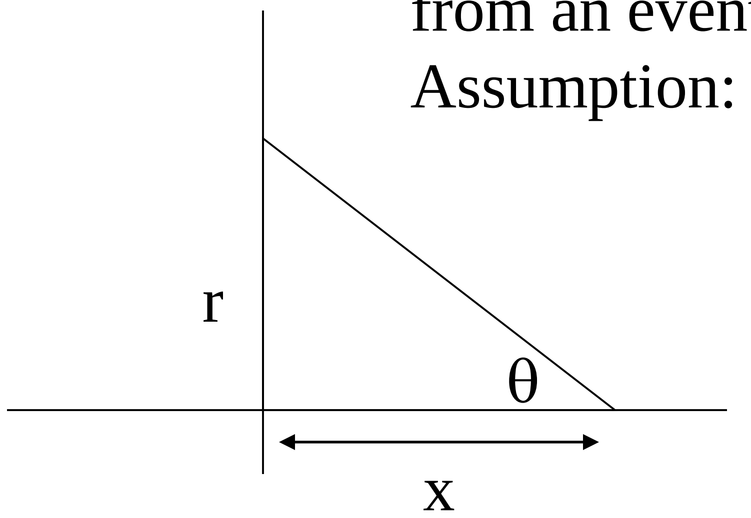
High density (& high z material (eg. Calcium Tangstate) photoelectric absorption

Screen creates light fluorescent photons. These get captured or trapped by silver bromide particles on film.

Analysis: First calculate spray of light photons from an event at depth x.

Assumption: An isotropic radiator

Uniform light propagation (neglect granular nature of phosphor)



invariant impulse response: $H(r) = h(0)\cos^3\theta$

Obliquity factor Cos θ and Inverse-square-law fall off $\cos^2\theta$

Since $\cos(\theta) = \dfrac{x}{\sqrt{(x^2 + r^2)}}$ $H(r) = h(0)\, x^3 / (x^2 + r^2)^{3/2}$

$h(0) = K/x^2$ Response at r=0 K= constant x^2 =inverse falloff

$$h(r) = K\frac{x}{(x^2 + r^2)^{3/2}}$$

Analysis in Frequency Domain

For space invariant system, the Frequency Response of circular function in polar coordinate is given by Hankel transform as:

$$H_1(r) = Ft\{h(r)\} = 2\pi \int_o^\infty \frac{Kx}{(x^2 + r^2)^{3/2}} \mathrm{J}_0(2\pi\rho r) r dr$$

$J_0\ (2\pi\rho r)r$ is kernel of Fourier Bessel transform of a circular symmetric function and ρ is radial spatial frequency

The resultant transform (from a table Hankel transforms) is:

$$H_1(\rho) = 2\pi K e^{-2\pi x \rho}$$

Normalize to DC Value, to eliminate constant terms:

$$H(\rho) = \frac{H_1(\rho)}{H_1(0)} = e^{-2\pi x \rho}$$

Notice this is the transfer function from a photon giving dits energy at depth x of screen.

Analysis in Frequency Domain

In order to find the average transfer function $\overline{H}(\rho)$ from a large number of photons, we integrate over the probability density *P(x)* (the likelihood of where events will occur in the scintillating material).

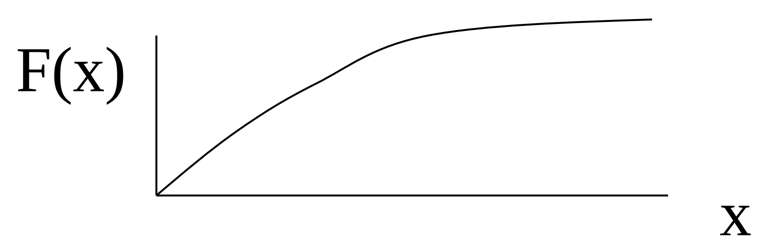
$$\overline{H}(\rho) = \int H(\rho)P(x)dx = \int e^{-2\pi x\rho} P(x)dx$$

Probability Density function can be determined from Distribution function *F(x)* as:

$$P(x) = \frac{dF(x)}{dx} = \mu e^{-\mu x}$$

Where Distribution function for an infinitey thick phosphor is:

$$F(x) = 1 - e^{-\mu x}$$



For a screen of thickness d:

$$F(x) = 1 - e^{-\mu x} / 1 - e^{-\mu d}$$

For only captured photons, Thus *F(x)* varies from 0 to 1 as *x* varies from 0 to *d*

Then:

$$p(x) = \frac{\mu e^{-\mu x}}{1 - e^{-\mu d}}$$

The Normalized spectrum from the large number photons is:

$$\overline{H}(\rho) = \int H(\rho) P(x) dx = \frac{1}{1 - e^{-\mu d}} \int_0^d e^{-2\pi x \rho} \mu e^{-\mu x} dx$$

$$\overline{H}(\rho) = \frac{\mu}{(2\pi\rho + \mu)(1 - e^{-\mu d})} [1 - e^{-d(2\pi\rho + \mu)}]$$

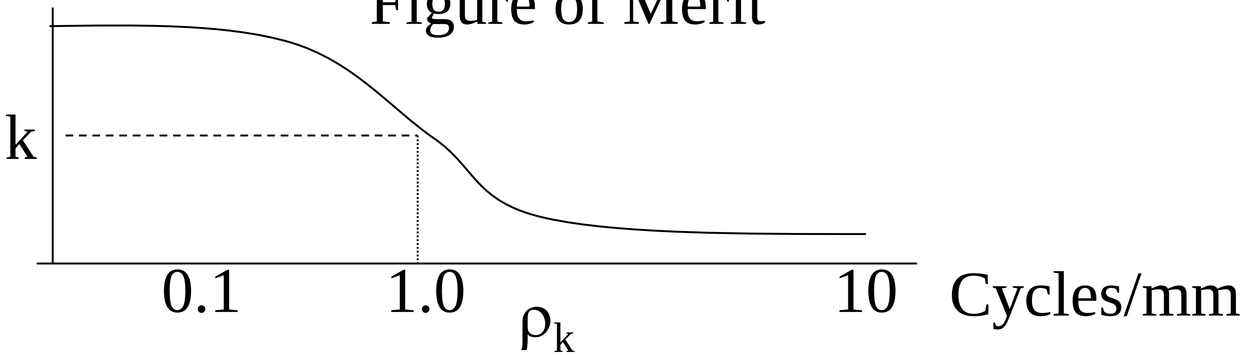
$$\overline{H}(\rho) = \frac{\mu}{(2\pi\rho + \mu)(1 - e^{-\mu d})}[1 - e^{-d(2\pi\rho+\mu)}]$$

We would like to describe a figure of merit that would describe a cutoff spatial frequency, or effective bandwidth.

For a typical calcium tungstate screen with d approximately 0.25 mm and μ=0.15/cm, the bracketed [] term can be approximated to 1 above some low spatial frequencies.

For example at ρ=1.0 cyc/mm: $\overline{H}(\rho) = 0.53$ and the bracket is 0.85.

Figure of Merit



For moderate k, (i.e. at a cutoff frequency)

$$\overline{H}(\rho) = k \cong \frac{\mu}{(2\pi\rho_k + \mu)(1 - e^{-\mu d})}$$

Let $(1 - e^{-\mu d}) = \eta$ the capture efficiency of the screen

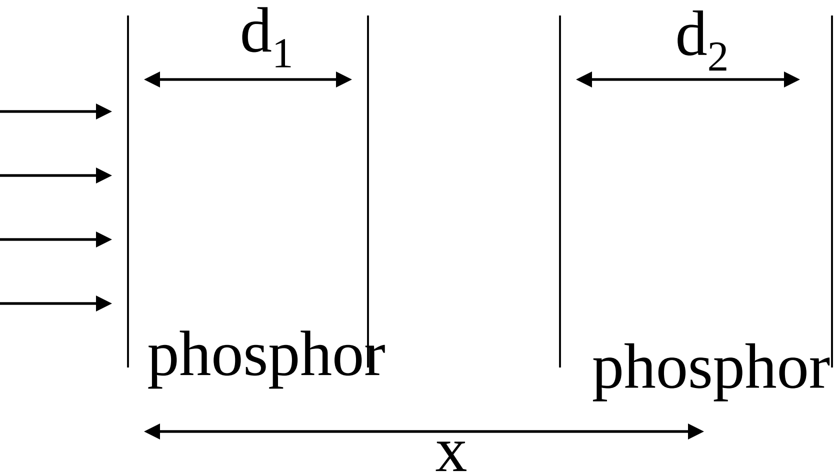
Then: $k \approx \mu / (2\pi p_k + \mu)\eta$ $2\pi k \eta p_k = (1-\eta k)\mu$ For $\eta k << 1$

$$p_k = \frac{\mu}{2\pi k \eta}$$

As the efficiency increases, ρ_k decreases. This is because η increases as d increases.

Dual Screen Systems

Dual Screen Systems



Double Emulsion film

Let $d_1 + d_2 = d$ so we can compare performance.

$H(\rho,x) = e^{-2\pi\rho (d_1 - x)}$ for $0 < x < d_1$

$H(\rho,x) = e^{-2\pi\rho (x - d_1)}$ for $d_1 < x < d_1 + d_2$

$$\overline{H}(\rho) = \frac{\mu}{(1-e^{-\mu d})}\left\{\int_0^{d_1} e^{-2\pi\rho(d_1-x)}e^{-\mu x)}dx + \int_{d_1}^{d_2} e^{-2\pi\rho(x-d_1)}e^{-\mu x)}dx\right\}$$

$$= \frac{\mu}{(1-e^{-\mu d})}\left[\frac{e^{-\mu d_1} - e^{-2\pi\rho d_1}}{2\pi\rho - \mu} + \frac{e^{-\mu d_1} - e^{-(2\pi\rho d_2 + \mu d)}}{2\pi\rho + \mu}\right]$$

Again lets determine a cutoff frequency of ρ_k for the response $H(\rho_k) = k$,
If we assume $d_1 \approx d_2 = d/2$, than we can neglect $e^{-2\pi\rho d_1}$, $e^{-2\pi\rho d_2}$ because they will be small even for relatively small spatial frequencies.
$e^{-\mu d}$ is also small compared to $e^{-\mu d_1}$

Since $(2\pi\rho)^2 >> \mu^2$ is true for all but lowest frequency, then:

$$\frac{e^{-\mu d_1}}{2\pi\rho - \mu} + \frac{e^{-\mu d_1}}{2\pi\rho + \mu} \approx \frac{2e^{-\mu d_1}}{2\pi\rho}$$

$$H(p_k) = k = \frac{\mu}{\eta}e^{-\mu d_1}\frac{2}{2\pi\rho_k}$$

$$\boxed{\rho_k = \frac{\mu}{2\pi k\eta}2e^{-\mu d_1}}$$

$$\boxed{\rho_k = \frac{\mu}{2\pi k\eta}(2e^{-\mu d_1})}$$

Compare this cutoff frequency to the single screen cutoff. The difference is the factor $2e^{-\mu d_1}$

Since

$$\eta = 1 - e^{-\mu d}$$

$$e^{-\mu d/2} = \sqrt{1-\eta}$$

$$So \cdots 2e^{-\mu d/2} = 2e^{-\mu d_1} = 2\sqrt{1-\eta}$$

Improvement is $2\sqrt{1-\eta}$

With $\eta \approx 0.3$, improvement is 1.7

Use improvement to lower dose, improve contrast, or some combination.

Overall Response

Assuming a circularly symmetric source,
Detector response is also circularly symmetric.

Intensity at detector

$I_d\ (x_d, y_d) = Kt\ (x_d/M, y_d/M) ** (1/m^2)\ s\ (r_d/m) ** h\ (r_d)$

Frequency Domain

Frequency component at detector:

$$I_d\,(u,v) = KM^2T\,(Mu,Mv)\,\underbrace{S(m\rho)\cdot H(\rho)}_{H_0(\rho)}$$

Frequencies components at Object of interest :

$$I_d\,(u/M,v/M) = KM^2T\,(u,v)\,S((m/M)p)\cdot H(p/M)$$

Product of 2 Low Pass Filters.

$$H_0(\rho) = S([\tfrac{1-z}{d}]\rho)H([\tfrac{z}{d}]\rho)$$

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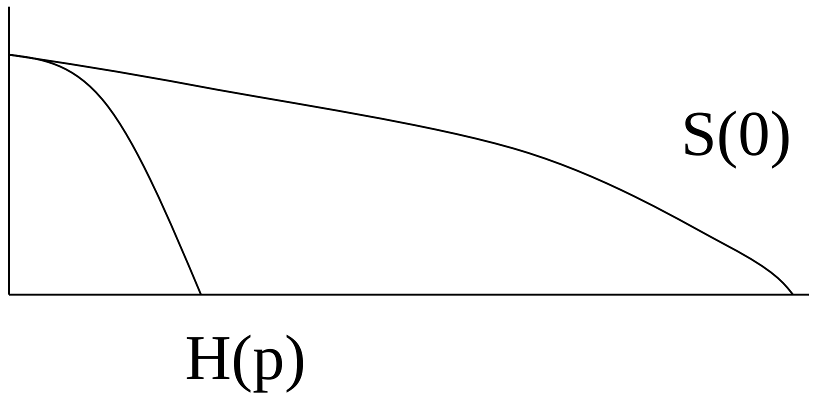
As z $\rightarrow$ d
Object close to Detector



As z $\rightarrow$ 0
Object close to source